DAA-003-001501 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

April / May – 2015

Physics : Paper 501

(Mathematical Physics, Classical Mechanics and Quantum mechanics)

Faculty Code : 003

Subject Code : 001501

Time : **2:30** Hours] [Total Marks : **70**

Instructions : (i) Give answer of all the questions in given answer sheet.
(ii) All questions are compulsory.
(iii) Symbols have their usual meaning.
(iv) Figure on right hand side indicates full mark.

1 Select true answer from the given options : **20**

(1) What is Fourier coefficient b_n ?

(A) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos mx\, dx$ (B) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\sin mx\, dx$

(C) $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin mx\, dx$ (D) $\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos mx\, dx$

(2) What are Fourier coefficient of a series ?

$\frac{h}{2}+\frac{2h}{\pi}\left(\sin\pi+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\ldots\right)$?

(A) $a_0=\frac{h}{2}, a_n=\frac{2h}{m\pi}, b_n=\frac{2}{m\pi}$

(B) $a_0=h,\ a_n=0, b_n=\frac{2h}{m\pi}$

(C) $a_0=\frac{1}{2},\ a_n=0, b_n=\frac{h}{m\pi}(1-\cos m\pi)$

(D) $a_0=\frac{h}{2},\ a_n=0, b_n=\frac{h}{m\pi}(1-\cos m\pi)$

(3) If *f(x)* is an odd function in interval $(-\pi,\pi)$, then what will be the $a_0 \cdot a_n$ and b_n ?

(A) $a_0 = 0,\ a_n = 0,\ b_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\sin mx\ dx$

(B) $a_0 = 0,\ a_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\cos mx\ dx,\ b_n = 0$

(C) $a_0 = 0,\ a_n = \frac{1}{\pi}\int_{0}^{\pi} f(x)\cos mx\ dx,\ b_n = 0$

(D) $a_0 = 0,\ a_n = 0,\ b_n = \frac{2}{\pi}\int_{0}^{\pi} f(x)\sin mx\ dx$

(4) What is Fourier coefficient b_n for a piecewise monotonic periodic function :

$F(x) = \frac{\pi^2}{12} - \frac{\pi^2}{4},\ for -\pi < x < \pi$?

(A) 1

(B) 0

(C) $\frac{\cos n\pi}{n^2}$

(D) $\frac{-3\cos n\pi}{n}$

(5) If $L = \frac{1}{2}(m_1 + m_2)x^2 + m_1 gx + m_2 g(l - x),\ then \frac{\partial L}{\partial x} = ?$

(A) $(m_1 + m_2)g$

(B) $(m_1 - m_2)g$

(C) $(m_1 + m_2)x$

(D) $(m_1 - m_2)x$

(6) Which equation of constraint belongs to class holonomic ?

(A) $F_i(r_{j,}\ t) = 0$

(B) $F_i(r_{j,}\ t) > 0$

(C) $F_i(r_{j,}\ t) < 0$

(D) $F_i(r_{j,}\ t) \neq 0$

(7) A bob moves on the surface of a sphere. The radius of the sphere is l. The bob is restricted to move in xy-plane only, what will be the generalized coordinates ?

(A) $q = \sin^{-1}\frac{y}{x}$ (B) $q = \cos^{-1}\frac{z}{l}$

(C) $q = \tan^{-1}\frac{y}{x}$ (D) $q = \tan^{-1}\frac{z}{l}$

(8) What is the Lagrange's equation of motion for a conservative holonomic system ?

(A) $\frac{d}{dt}.\frac{\partial L}{\partial q_j} - \frac{\partial L}{\partial q_j} = Q$ (B) $\frac{d}{dt}.\frac{\partial L}{\partial q_j} - \frac{\partial L}{\partial q_j} = 0$

(C) $\frac{d}{dt}.\frac{\partial T}{\partial q_j} - \frac{\partial T}{\partial q_j} = Q$ (D) $\frac{d}{dt}.\frac{\partial V}{\partial q_j} - \frac{\partial V}{\partial q_j} = Q$

(9) What is the equation of constraint of a simple pendulum, if (r,θ) be the coordinates of the bob and 1 be the length of the pendulum ?

(A) $T = \frac{1}{2}mr^2$ (B) $V = mgr\cos\theta$

(C) $L = T - V$ (D) $r = l$

(10) If $H = 2ml\left(\theta p_z - mlz\sin\theta\right)\sin\theta - mgz);\ then \frac{\partial H}{\partial \theta} = ?$

(A) - mg (B) $2ml\theta \sin\theta$

(C) $2mlp_z \sin\theta$ (D) $2ml\sin\theta$

(11) A particle move from $r_1(t_1)$ to $r_2(t_2)$. According to Hamilton's principle, work done in passing from the true path to the virtual path is $\delta w = F.\delta r$. The total force F acting on the particle is the vector sum of the applied force F_a and the force of constraint f_c. The varied consider here is such that no work is done by the force of constraint. With which comment you may agree ?

(A) Force of constraint and the displacement consider to obtain variation in path are at zero angle to each other.

(B) Force of constraint and the displacement consider to obtain variation in path are at 45 angle to each other.

(C) Force of constraint and the displacement consider to obtain variation in path are at 180 angle to each other.

(D) Force of constraint and the displacement consider to obtain variation in path are at right angle to each other.

(12) An electrical circuit containing an inductance L, resistance R and capacitance C in series with an external electromotive force $\in(t)$. Which statement is relevant with it ?

(A) Voltage across each element is same.

(B) Current flowing through R, L and C are different.

(C) emf $\in(t)$ is equal to the sum of voltage across each elements.

(D) Current through R and L is finite but through C is zero.

(13) What is the Hamilton's equation of motion ?

(A) $\frac{\partial L}{\partial t} = 0, \frac{\partial H}{\partial t} = 0$

(B) $\frac{\partial H}{dt} = \frac{\partial H}{\partial t}$

(C) H = constant

(D) $q_k = \frac{\partial H}{\partial P_K}, p_k = -\frac{\partial H}{\partial q_k}$

(14) A particle limited to the x-axis has the wave function $\psi = ax$ between $x = 0$ and $x = 1$. $\psi = 0$ elsewhere, what is the expectation value, $< x >$, of the particle's position ?

(A) $\frac{a^3}{3}$ (B) $\frac{a^2}{4}$

(C) $\frac{a^2}{2}$ (D) $\frac{a^4}{4}$

(15) The kinetic energy is given as $T = \frac{p^2}{2m}$. What is the kinetic energy operator ?

(A) $i\hbar\frac{\partial}{\partial t}$ (B) $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$

(C) $\frac{\hbar}{i}\frac{\partial}{\partial x}$ (D) $\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial t^2}$

(16) $\left[p, x^3\right] = ?$

(A) $-3i\hbar x$ (B) $-3i\hbar x^2$

(C) $-3i\hbar x^3$ (D) $-3i\hbar x^4$

(17) When a wave function ψ is said to be normalized ?

(A) $\int_0^\infty |\psi|^2 d^3r = 0$ (B) $\int_{-\infty}^0 |\psi|^2 d^3r = 0$

(C) $\int_{-\infty}^\infty |\psi|^2 d^3r = 1$ (D) $\int_{-\infty}^\infty |\psi|^2 d^3r \neq 1$

(18) Which of the following is not appropriate to the wave function ?

(A) ψ must be continuous

(B) ψ must be single valued everywhere

(C) ψ must be non normalizable

(D) ψ must be normalizable

(19) A particle is moving inside a infinite square well potential, its position x at any instant is given by $-L < x < 0$, what is its potential function ?

(A) $V = \infty, for - L < x < 0, V = 0, for x \geq 0, V = 0, for x \leq -L,$

(B) $V = 0, for - L < x < 0, V = \infty, for x \geq 0, V = 0, for x \leq -L,$

(C) $V = 0, for - L < x < 0, V = 0, for x \geq 0, V = \infty, for x \leq -L,$

(D) $V = 0, for - L < x < 0, V = \infty, for x \geq 0, V = \infty, for x \leq -L,$

(20) $\left[x, p_y\right] = ?$

(A) $\left[x, p_y\right]$ (B) $\left[y, p_x\right]$

(C) $\left[z, p_z\right]$ (D) $\left[y, p_z\right]$

2 (A) Answer any **three** in brief : **6**

(1) $f(x) = 2x$, for $0 \leq x \leq \pi$ and $f(x) = x$, $-\pi \leq x \leq 0$, is it odd or even ?

(2) Give illustrations of holonomic and non-holonomic constraints.

(3) What is called cyclic coordinate ?

(4) $\nabla \cdot B = 0$ and $\nabla \times E = 0$, then what will be the B and E ?

(5) State Hamilton's principle.

(6) Why the multipliers called undetermined in Lagrangian formulation ?

2 (B) Answer any **three** : **9**

(1) Obtain cosine series.

(2) A piecewise monotonic periodic function $f(x)$ with period 2π is defined as follow :

$$F(x) = x^2, -\pi \leq x \leq \pi$$

Determine the Fourier coefficient a_0.

(3) Obtain D' Alembert's Principle.

(4) Derive generalized coordinates of a particle constrained to move on the surface of a sphere.

(5) What is configuration space ?

(6) Show that Lagrange's and Newton's equations are equivalent.

2 (C) Answer any **two** in detail. **10**

(1) Expand $F(x) - x\sin x$, for $-\pi \leq x \leq \pi$, in Fourier series.

$F(x)$ is piecewise monotonic and periodic function.

(2) Expand the following function in Fourier series in the interval $(-\pi, \pi)$.

$$f(x) = -\frac{\pi + x}{2} \quad \text{for} \quad -\pi < x < 0$$

$$= \frac{\pi + x}{2} \quad \text{for} \quad 0 < x < \pi$$

(3) Expand the function $y = \cos 2x$ in a series of sines in the interval of $(0, \pi)$.

(4) Obtain Lagrange's equations of motion.

(5) Explain velocity dependent potential of electromagnetic field.

3 (A) Answer any **three** in brief : **6**

(1) What is the time dependent form of Schrodinger's equation in three dimensions ?

(2) Explain operator correspondence.

(3) When a wave function ψ is said to be normalized ?

(4) A wave function $\psi(x) = A.e^{ikx}$ is normalized over a region $-a \leq x \leq a$, then A = ?

(5) Evaluate : $\left[x, p^n\right]$.

(6) Show that product of two self-adjoint operators is not necessarily self-adjoint.

3 (B) Answer any **three** : **9**

(1) Write fundamental postulates of wave mechanics.

(2) Show that $(A+B)^{\dagger} = A^{\dagger} + B^{\dagger}$

(3) Give general features of a particle in a square well potential.

(4) What is expectation value ? Give expression in case of a non-normalized wave function.

(5) A particle limited to the x-axis has the wave function $\psi = ax$ between $x=0$ and $x=1$; $\psi = 0$ elsewhere, what is the probability that the particle can be found between $x=0.45$ and $x=0.55$?

(6) Explain the penetration into classical forbidden region in square well potential.

3 (C) Answer any **two** in detail : **10**

(1) Explain even and odd parity of an Eigen functions in case of a square well.

(2) Show that the Schrodinger's wave equation leads to the satisfaction of classical (Newton's) law of motion on the average.

(3) Describe theory of Hamilton's equation with help of a simple pendulum with moving support.

(4) Prove that $\left[L_z, L_x\right] = i\hbar L_y$.

(5) Describe Eigen value problem and degeneracy.